

Factorization and hadronic B decays in the heavy quark limit

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Abstract. Recent theoretical investigations (M. Beneke *et al* 1999 *Phys. Rev. Lett.* **83** 1914) of two-body hadronic B decays have provided justification, at least to order α_s , for the use of the factorization ansatz to evaluate the B decay matrix elements provided the decay meson containing the spectator quark is not heavy. Although data is now available on many charmless two-body B decay channels, it is so far not very precise with systematic and statistical errors in total of the order of 25% or greater. Analyses have been made on $\pi\pi$ and πK channels with somewhat contradictory results (M. Beneke *et al* 2001 *Nucl. Phys. B* **606** 245; M. Ciuchini *et al* 2001 hep-ph/0110022). We take an overview of these channels and of other channels containing vector mesons. Because factorization involves many poorly known soft QCD parameters, and because of the imprecision of current data, we present simplified formulae for a wide range of B decays into the lowest mass pseudoscalar and vector mesons. These formulae, valid in the heavy quark limit, involve a reduced set of soft QCD parameters and, although resulting in some loss of accuracy, should still provide an adequate and transparent tool with which to confront data for some time to come. Finally we confront these formulae with data on nineteen channels. We find a plausible set of soft QCD parameters that, apart from three pseudoscalar vector channels, fit the branching ratios and the recently measured value of $\sin(2\beta)$ quite well.

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1. Introduction

Much experimental effort is being expended in the study of B meson decays [1, 2, 3, 4, 5, 6] and the next decade will see intensive investigation of the B -meson system at the Tevatron, the SLAC and KEK B -factories, and at the LHC. The aim is to establish the Cabbibo-Kobayashi-Maskawa (CKM) parameters to an accuracy that will test the consistency of the Standard Model (SM) description of CP asymmetry, hopefully to a precision comparable to that of other aspects of the Standard Model. B meson decays should have many channels which exhibit CP asymmetry but even with precise data

there will be a problem of reliably unravelling the underlying weak decay mechanisms from the distortions caused by the strong interactions.

Hard QCD corrections to the underlying b quark weak decay amplitudes involve gluon virtualities between the electroweak scale M_W and the scale $O(m_b)$ and are implemented through renormalization of the calculable short distance Wilson coefficients C_i in the low energy effective weak Hamiltonian[7] for $\Delta B = 1$ decays at scale $\mu = O(m_b)$

$$H_{\text{eff}}(\mu) = \frac{G_F}{\sqrt{2}} \left\{ \sum_{p=u,c} \lambda_p [C_1(\mu)O_1^p + C_2(\mu)O_2^p] - \lambda_t \sum_{i=3,\dots,6} C_i(\mu)O_i \right\} + \text{other terms} \quad (1)$$

where $\lambda_p \equiv V_{pq}^* V_{pb}$ is a product of CKM matrix elements, $q = d, s$ and the local $\Delta B = 1$ four-quark operators are

$$\begin{aligned} O_1^p &\equiv (\bar{q}_\alpha p_\alpha)_{V-A} (\bar{p}_\beta b_\beta)_{V-A}, \\ O_2^p &\equiv (\bar{q}_\alpha p_\beta)_{V-A} (\bar{p}_\beta b_\alpha)_{V-A}, \\ O_{3,5} &\equiv (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V\mp A} \\ O_{4,6} &\equiv (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V\mp A}, \end{aligned} \quad (2)$$

where $q' \in \{u, d, s, c\}$, α and β are colour indices, and we have used the notation, for example,

$$(\bar{q}_\alpha b_\alpha)_{V-A} (\bar{q}'_\beta q'_\beta)_{V\mp A} = [\bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha] [\bar{q}'_\beta \gamma^\mu (1 \mp \gamma_5) q'_\beta]. \quad (3)$$

The operators in (2) are associated with particular processes (see [7] for a detailed discussion): $O_{1,2}$ are the tree current-current operators and $O_{3,\dots,6}$ are QCD penguin operators. The “other terms” indicated in (1) are small in the SM. They include magnetic dipole transition operators and electroweak penguin operators. The most important of these is the electroweak penguin operator

$$O_9 \equiv \frac{3}{2} (\bar{q}_\alpha b_\alpha)_{V-A} \sum_{q'} e_{q'} (\bar{q}'_\beta q'_\beta)_{V-A} \quad (4)$$

where $e_{q'} = 2/3$ ($-1/3$) for $u(d)$ -type quarks. The Wilson coefficient C_9 is larger than the QCD penguin coefficients C_3 and C_5 and contributes $-(G_F/\sqrt{2})\lambda_t C_9(\mu)O_9$ to $H_{\text{eff}}(\mu)$.

Inclusion of strong interaction effects below the scale μ is a very difficult task involving, for the two-body hadronic decay $B \rightarrow h_1 h_2$, the computation of the matrix elements $\langle h_1 h_2 | O_i | B \rangle$. Until recently, most theoretical studies of two-body hadronic decay invoked the factorization approximation [8] in which final state interactions are neglected and $\langle h_1 h_2 | O_i | B \rangle$ is expressed as a product of two hadronic currents: $\langle h_1 | J_{1\mu} | B \rangle \langle h_2 | J_2^\mu | 0 \rangle$. The operators O_2^p and $O_{4,6}$ are Fierz transformed into a combination of colour singlet-singlet and octet-octet terms and the octet-octet terms then discarded. The singlet-singlet current matrix elements are then expressed in terms of known decay rates and form factors. Consequently, the hadronic matrix elements are expressed in terms of the combinations

$$a_{2i-1} = C_{2i-1} + \frac{1}{N_c} C_{2i}, \quad a_{2i} = C_{2i} + \frac{1}{N_c} C_{2i-1} \quad (5)$$

where $i = 1, 2, 3$ and $N_c = 3$ is the number of colours.

In the widely used so-called “generalized factorization” approach [9, 10, 11, 12], the renormalization scale dependence of the hadronic matrix elements $\langle h_1 h_2 | O_i | B \rangle$, lost through factorization, is compensated for through the introduction of effective Wilson coefficients $C_i^{\text{eff}}(\mu)$ such that

$$C_i(\mu) \langle O_i(\mu) \rangle = C_i^{\text{eff}}(\mu) \langle O_i \rangle^{\text{tree}}. \quad (6)$$

The effective Wilson coefficients $C_i^{\text{eff}}(\mu)$, $i = 3, \dots, 6$ for the QCD penguins depend upon the gluon momentum q^2 and generate strong phases as q^2 crosses the $u\bar{u}$ and $c\bar{c}$ thresholds [13]. The neglected octet-octet terms are compensated for by replacing N_c by a universal free parameter ξ . The assumed universal a_i parameters are then determined by fitting to as much data as possible. Some authors [12] have allowed the ξ parameter for the $(V - A) \otimes (V - A)$ and $(V - A) \otimes (V + A)$ contributions to be different.

Recently there has been significant progress in the theoretical understanding of hadronic decay amplitudes in the heavy quark limit [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24]. These approaches, known as QCD (improved) factorization, exploit the fact that m_b is much greater than the QCD scale Λ_{QCD} and show that the hadronic matrix elements have the form

$$\langle h_1 h_2 | O_i | B \rangle = \langle h_1 | J_{1\mu} | B \rangle \langle h_2 | J_2^\mu | 0 \rangle \left[1 + \sum_n r_n \alpha_s^n + \mathcal{O}(\Lambda_{\text{QCD}}/m_b) \right] \quad (7)$$

provided the spectator quark does not go to a heavy meson. If the power corrections in Λ_{QCD} and radiative corrections in α_s are neglected, conventional or “naive” factorization is recovered. Although naive factorization is broken at higher order in α_s , these non-factorizable contributions can be calculated systematically.

For $B \rightarrow h_1 h_2$ in which both h_1 and h_2 are light, and h_2 is the meson that does not pick up the spectator quark, [14, 15, 23] find that all non-factorizable contributions are real and dominated by hard gluon exchange which can be calculated perturbatively, and all leading order non-perturbative soft and collinear effects are confined to the $B - h_1$ system and can be absorbed into form factors and light cone distribution amplitudes. Strong rescattering phases are either perturbative or power suppressed in m_b and, at leading order, arise via the Bander-Silverman-Soni mechanism [25] from the imaginary parts of the hard scattering kernels in r_1 . The contribution from the annihilation diagram (in which the spectator quark annihilates with one of the b decay quarks) is found to be power suppressed. In contrast to one of the basic assumptions of generalized factorization, the corrections to naive factorization are process dependent and have a richer structure than merely allowing ξ to be different for the $(V - A) \otimes (V - A)$ and $(V - A) \otimes (V + A)$ contributions.

Similar in many ways to QCD factorization is the hard scattering or perturbative QCD approach [26]. However, here it is argued that all soft contributions to the $B - h_1$ form factor are negligible if transverse momentum with Sudakov resummation is included, so that the form factors can be perturbatively calculated. Also, it is argued that Sudakov suppression of long distance effects in the B meson is needed to control

higher order effects in the Beneke *et al* [14, 15] approach. Non-factorizable contributions are now found to be complex but the imaginary parts are much smaller than that of the (factorizable) contribution from the annihilation diagram. Several aspects of the perturbative QCD approach have been strongly criticized [23, 27].

In summary, recent theoretical investigations have provided justification, at least to order α_s , for invoking factorization to evaluate the hadronic matrix elements $\langle h_1 h_2 | O_i | B \rangle$ provided the decay meson containing the spectator quark is not heavy. Unfortunately, the numerical results from the QCD factorization calculations are quite sensitive to the input assumptions on the quark distribution functions [28], and the contributions from the logarithmic end point divergences in hard spectator scatterings and weak annihilation [24], thus giving rise to significant theoretical uncertainties which currently limit their usefulness.

The success claimed [23] for QCD improved factorization in fitting the observed branching ratios for $B \rightarrow \pi\pi$ and $B \rightarrow K\pi$ has been queried by Ciuchini *et al* [29] who argue that, with the CKM angle γ constrained by the unitarity triangle analysis of [30], non-perturbative Λ_{QCD}/m_b corrections, so called “charming penguins”, are important where the factorized amplitudes are either colour or Cabibbo suppressed. Factorization at leading order in Λ_{QCD}/m_b cannot reproduce the observed $B \rightarrow \pi\pi$ and $B \rightarrow K\pi$ decays and must be supplemented with enhanced c -loop penguin diagrams.

In this paper we present formulae based on the factorization ansatz for B decay into the lowest mass pseudoscalar and vector mesons. We focus on the heavy meson limit in which combinations of the different parameters that characterize the current matrix elements reduce to just one. This is a consistent approach within the heavy quark approximation and we believe that the resulting more simple formulae will provide an adequate and more transparent tool with which to confront data for some time to come. Finally we confront these formulae with data on nineteen channels. With $\sin(2\beta)$ as measured by the BaBar [3] and Belle collaborations [6] we find a plausible set of soft QCD parameters that, apart from three pseudoscalar vector channels, fit the data well.

2. Factorized decay amplitude

The amplitude for charmless hadronic b decays for all these factorization schemes can be written as

$$\langle h_1 h_2 | H^{\text{eff}} | B \rangle = \frac{G_F}{\sqrt{2}} \sum_{i=1,\dots,6,9} p_i [Q_i(h_1, h_2) + Q_i(h_2, h_1)] \quad (8)$$

where

$$\begin{aligned} Q_1(h_1, h_2) &= \langle h_1 | (\bar{u}_\alpha b_\alpha)_{V-A} | B \rangle \langle h_2 | (\bar{q}_\beta u_\beta)_{V-A} | 0 \rangle, \\ Q_2(h_1, h_2) &= \langle h_1 | (\bar{q}_\alpha b_\alpha)_{V-A} | B \rangle \langle h_2 | (\bar{u}_\beta u_\beta)_{V-A} | 0 \rangle, \\ Q_3(h_1, h_2) &= \langle h_1 | (\bar{q}_\alpha b_\alpha)_{V-A} | B \rangle \langle h_2 | (\bar{q}'_\beta q'_\beta)_{V-A} | 0 \rangle, \\ Q_4(h_1, h_2) &= \langle h_1 | (\bar{q}'_\alpha b_\alpha)_{V-A} | B \rangle \langle h_2 | (\bar{q}_\beta q'_\beta)_{V-A} | 0 \rangle, \\ Q_5(h_1, h_2) &= \langle h_1 | (\bar{q}_\alpha b_\alpha)_{V-A} | B \rangle \langle h_2 | (\bar{q}'_\beta q'_\beta)_{V+A} | 0 \rangle, \end{aligned}$$

$$\begin{aligned}
Q_6(h_1, h_2) &= -2 \langle h_1 | (\bar{q}'_\alpha b_\alpha)_{S-P} | B \rangle \langle h_2 | (\bar{q}_\beta q'_\beta)_{S+P} | 0 \rangle, \\
Q_9(h_1, h_2) &= \langle h_1 | (\bar{q}_\alpha b_\alpha)_{V-A} | B \rangle \langle h_2 | e_{q'} (\bar{q}'_\beta q'_\beta)_{V-A} | 0 \rangle,
\end{aligned} \tag{9}$$

with

$$(\bar{q}'_\alpha b_\alpha)_{S-P} = \bar{q}'_\alpha (1 - \gamma_5) b_\alpha, \quad (\bar{q}_\beta q'_\beta)_{S+P} = \bar{q}_\beta (1 + \gamma_5) q'_\beta, \tag{10}$$

and

$$p_{1,2} \equiv \lambda_u a_{1,2}; \quad p_{3,\dots,6,9} \equiv \lambda_u a_{3,\dots,6,9}^u + \lambda_c a_{3,\dots,6,9}^c. \tag{11}$$

The form of the matrix element Q_6 is a consequence of a Fierz transformation on the $(V-A) \otimes (V+A)$ term. An example of a process in which h_1 (and h_2) can attach itself to either current is for $\bar{q}' = \bar{q} = d$ where the matrix element for $\bar{B}^0 \rightarrow \pi^0 \rho^0$ will have contributions from both terms.

The matrix elements in Q_6 can be estimated from the matrix elements of the electroweak currents by taking the quarks, at some appropriate mass scale, to be on mass shell. For the current

$$J^\mu = \bar{q}_1 \gamma^\mu (1 \mp \gamma_5) q_2 \tag{12}$$

this yields

$$i \partial_\mu J^\mu = (m_2 - m_1) \bar{q}_1 q_2 \pm (m_1 + m_2) \bar{q}_1 \gamma_5 q_2. \tag{13}$$

Forming matrix elements of (13) between scalar, pseudoscalar and vector states as appropriate, one of the terms on the right hand side will be identically zero and the other will then be determined by the matrix element of the left hand side.

The a_i coefficients in (8) have the form

$$a_i = a_i^{\text{LO}} + \alpha_s a_i^{(1)} \tag{14}$$

where the leading order (LO) a_i^{LO} are given by the naive factorization expressions (5) with the Wilson coefficients C_i taken at next to leading order (NLO). Detailed expressions for the $a_i^{(1)}$ are given in [14, 15, 18, 20, 22, 23] for $B \rightarrow PP$, in [19] for $B \rightarrow PV$ and in [21] for $B \rightarrow VV$, where P and V denote light pseudoscalar and vector mesons respectively. Note that [23] claim there are errors in the expressions given by [18, 20, 22] for $B \rightarrow PP$.

The beauty of the result (14) is that the difference between the decay rate formulae of naive and generalized factorization as presented in numerous works [9, 10, 12] and that based upon QCD factorization lies only in the coefficients a_i , the factorization matrix elements Q_i are common to all these approaches. Also, if the soft gluon physics is all accounted for in the current matrix elements then, in the heavy quark limit, N_c should be taken to be three.

The corrections to the coefficients a_i so far presented are to first order in α_s . An encouraging feature is that they are not generally large, which leads one to hope that the precision with which the SM can be tested will be determined by the proximity of the b quark mass to the heavy quark limit and the precision of our knowledge of the soft QCD parameters, B meson semileptonic transition form factors and meson light cone

distribution amplitudes. An example of the differences in the a_i coefficients for several theoretical approaches is shown in Table 1. Here we compare the a_i coefficients for QCD factorization, generalized factorization and a simple tree plus penguin model at scale M_W . In this simple model, for example, the process $b \rightarrow dq\bar{q}$ produces a $q\bar{q}$ pair in a colour octet state so that they do not materialize in a $q\bar{q}$ meson; the \bar{q} must pair with the d quark and the q quark with the spectator antiquark so that $a_3 = a_5 = 0$ and $a_4 = a_6$. We have estimated these coefficients by assuming a quark parton distribution function $6x(1-x)$ in both h_1 and h_2 and that, for a given x_1 and x_2 , the $q\bar{q}$ pair has, neglecting quark masses, an invariant mass squared $q^2 = x_1x_2M_B^2$. The coefficient $a_4(=a_6)$ was then calculated by integrating the penguin amplitude given in [31]. All estimates of the penguin amplitudes and the associated a_i coefficients are uncertain; they involve the strong coupling α_s over a distribution in q^2 , models of quarks in hadrons and soft QCD parameters, some of which are only poorly known. Because of these uncertainties, to be confident of any conclusions drawn about the SM it will be important to have a consistent picture of as many channels of charmless B decay as possible.

3. Reduction of factorization matrix elements

The factorization matrix elements Q_i in (9) involve products of current matrix elements which are evaluated through the introduction of numerous soft QCD parameters such as meson decay constants and transition form factors. Explicit expressions given in the literature [9, 10, 19] for $B \rightarrow PP$, $B \rightarrow PV$ and $B \rightarrow VV$ decay amplitudes in the factorization approximation are extremely cumbersome and, consequently, are unlikely in the short term to be of great assistance to experimentalists in analyzing their limited data sets. We argue that the expressions for these factorized decay amplitudes can be simplified, albeit at some loss of accuracy, such that a wide range of decays can be expressed in terms of a relatively small number of soft QCD parameters which are, or will be, relatively well known. Our approximations consist of (1) neglect of terms $(m_{P,V}/m_B)^2$ in the decay amplitudes and rates and (2) use of universal generalized-factorization model a_i coefficients rather than the (slightly) process-dependent QCD-factorization model a_i coefficients. The effect of this latter approximation is considered in section 5 for decays involving π and K mesons for which the QCD factorization a_i coefficients are well established. The loss of accuracy incurred in our approach should not be significant until more precise data is available. Our expressions are formally exact in the heavy quark limit.

The π and K meson decay constants are defined through the matrix elements

$$\langle \pi^- | \bar{d}_\alpha \gamma_5 \gamma^\mu u_\alpha | 0 \rangle = -f_\pi p_\pi^\mu, \quad \langle K^- | \bar{s}_\alpha \gamma_5 \gamma^\mu u_\alpha | 0 \rangle = -f_K p_K^\mu. \quad (15)$$

Isospin symmetry then determines that

$$\langle \pi^0 | \bar{u}_\alpha \gamma_5 \gamma^\mu u_\alpha | 0 \rangle = -\langle \pi^0 | \bar{d}_\alpha \gamma_5 \gamma^\mu d_\alpha | 0 \rangle = -\frac{1}{\sqrt{2}} f_\pi p_\pi^\mu, \quad (16)$$

with similar relations for the K meson. The magnitudes of f_π and f_K are well determined from experimental measurements of the leptonic decays such as

$$\Gamma(\pi^- \rightarrow l^- \bar{\nu}) = \frac{G_F^2}{8\pi} |V_{ud}|^2 f_\pi^2 m_\pi m_l^2 \left[1 - \left(\frac{m_l}{m_\pi} \right)^2 \right]^2. \quad (17)$$

With an appropriate phase convention on the particle states, the decay constants can be taken to be real positive numbers which have the values $f_\pi = (0.1307 \pm 0.00046)$ GeV and $f_K = (0.1598 \pm 0.00184)$ GeV. Insofar as parity is a good quantum number, the vector current matrix elements for π and K are zero. By contrast, for the vector mesons ρ , ω , K^* and ϕ , the axial vector matrix elements are zero. The vector meson decay constants are defined by

$$\begin{aligned} \langle \rho^- | \bar{d}_\alpha \gamma^\mu u_\alpha | 0 \rangle &= f_\rho m_\rho \epsilon^\mu, & \langle K^{*-} | \bar{s}_\alpha \gamma^\mu u_\alpha | 0 \rangle &= f_{K^*} m_{K^*} \epsilon^\mu, \\ \langle \phi | \bar{s}_\alpha \gamma^\mu s_\alpha | 0 \rangle &= f_\phi m_\phi \epsilon^\mu, \\ \langle \omega | \bar{u}_\alpha \gamma^\mu u_\alpha | 0 \rangle &= \langle \omega | \bar{d}_\alpha \gamma^\mu d_\alpha | 0 \rangle = \frac{1}{\sqrt{2}} f_\omega m_\omega \epsilon^\mu \end{aligned} \quad (18)$$

where ϵ^μ is the meson polarization vector. Isospin symmetry determines the other matrix elements, for example

$$\langle \rho^0 | \bar{u}_\alpha \gamma^\mu u_\alpha | 0 \rangle = \frac{1}{\sqrt{2}} f_\rho m_\rho \epsilon^\mu. \quad (19)$$

The magnitudes of some vector decay constants can be inferred from measurements of τ lepton decay, for example

$$\Gamma(\tau^- \rightarrow K^{*-} \nu_\tau) = \frac{G_F^2}{16\pi} f_{K^*}^2 m_\tau^3 |V_{us}|^2 \left[1 - \left(\frac{m_{K^*}}{m_\tau} \right)^2 \right]^2 \left[1 + 2 \left(\frac{M_{K^*}}{m_\tau} \right)^2 \right] \quad (20)$$

and others from the meson decay rates into e^+e^- pairs:

$$\begin{aligned} \Gamma(\rho^0 \rightarrow e^+ e^-) &= \frac{2\pi}{3} \alpha^2 \frac{f_\rho^2}{m_\rho}, \\ \Gamma(\omega \rightarrow e^+ e^-) &= \frac{1}{9} \frac{2\pi}{3} \alpha^2 \frac{f_\omega^2}{m_\omega}, \\ \Gamma(\phi \rightarrow e^+ e^-) &= \frac{2}{9} \frac{2\pi}{3} \alpha^2 \frac{f_\phi^2}{m_\phi}. \end{aligned} \quad (21)$$

With a phase convention that makes the decay constants real and positive, these yield the well determined values $f_\rho = (0.216 \pm 0.005)$ GeV, $f_\omega = (0.194 \pm 0.004)$ GeV, $f_\phi = (0.233 \pm 0.004)$ GeV and $f_{K^*} = (0.216 \pm 0.010)$ GeV.

We now consider the B transition form factors. For pseudoscalar mesons, these are usually expressed in terms of two form factors $F_0(t)$ and $F_1(t)$:

$$\langle P | \bar{q}_\alpha \gamma^\mu b_\alpha | B \rangle = \left[(p_B + p_P)^\mu - \frac{(m_B^2 - m_P^2)}{t} q^\mu \right] F_1(t) + \frac{(m_B^2 - m_P^2)}{t} q^\mu F_0(t). \quad (22)$$

Here $q^\mu \equiv (p_B - p_P)^\mu$ and $t \equiv q^2$. The axial vector matrix elements are all zero. Both F_0 and F_1 are analytical functions of t with no singularity at $t = 0$. As there is no singularity in the matrix element, we have the constraint $F_0(0) = F_1(0)$. The nearest

singularity is for t real and greater than m_B^2 , distant from $t = 0$. Both F_0 and F_1 are often taken as simple pole or dipole dominant. For example, from lattice QCD,

$$F_1(t) = \frac{F(0)}{(1 - t/m^2)^2}, \quad F_0(t) = \frac{F(0)}{(1 - t/m^2)} \quad (23)$$

with $m^2 > m_B^2$. A virtue of the rather clumsy parameterization in (22) is that when a contraction is taken with the matrix element for the second decay meson then only F_0 contributes if the second meson is also a pseudoscalar and only F_1 contributes if the second meson is a vector. For example,

$$\langle \pi^- | \bar{d}_\alpha \gamma_5 \gamma^\mu u_\alpha | 0 \rangle \langle P | \bar{q}_\beta \gamma_\mu b_\beta | B \rangle = -f_\pi F_0(m_\pi^2)(m_B^2 - m_P^2), \quad (24)$$

$$\langle \rho^- | \bar{d}_\alpha \gamma^\mu u_\alpha | 0 \rangle \langle P | \bar{q}_\beta \gamma_\mu b_\beta | B \rangle = 2|p_\rho| m_B f_\rho F_1(m_\rho^2) \quad (25)$$

where the momentum of the ρ is given by

$$|p_\rho| = \left[\left(\frac{m_B^2 + m_\rho^2 - m_P^2}{2m_B} \right)^2 - m_\rho^2 \right]^{1/2}. \quad (26)$$

We have used $\epsilon \cdot p_B = |p_\rho| m_B / m_\rho$ in the B rest frame and taken the ρ meson to be moving along the z axis with zero helicity so that

$$m_\rho \epsilon_\rho^\mu = (p_\rho, 0, 0, E_\rho). \quad (27)$$

If terms in $(m_\pi/m_b)^2 = 0.0007$, $(m_\rho/m_B)^2 = 0.0212$, e.t.c. are neglected, as is appropriate for the heavy quark limit, then, because of the analytic structure of the form factors and the constraint $F_0(0) = F_1(0)$, we can write

$$\langle \pi^- | (\bar{d}_\alpha u_\alpha)_{V-A} | 0 \rangle \langle P | (\bar{q}_\beta b_\beta)_{V-A} | B \rangle = -f_\pi m_B^2 F_1(0) \quad (28)$$

and

$$\langle \rho^- | (\bar{d}_\alpha u_\alpha)_{V-A} | 0 \rangle \langle P | (\bar{q}_\beta b_\beta)_{V-A} | B \rangle = f_\rho m_B^2 F_1(0). \quad (29)$$

Apart from the well determined f_π and f_ρ , in the heavy quark limit the $B \rightarrow PP$ and the $B \rightarrow PV$ transitions through the term (29) are characterized by a single parameter. The use of two parameters is not consistent with the heavy quark limit.

The magnitudes of some form factors are measurable in principle through semileptonic decays such as $\bar{B}^0 \rightarrow \pi^+ + l^- + \bar{\nu}_e$, corresponding to $P = \pi^+$, $\bar{q} = \bar{u}$, for which, neglecting terms proportional to the lepton mass,

$$\frac{d\Gamma}{dt} = \frac{G_F^2}{24\pi^3} |V_{ub}|^2 |p_\pi| |F_1(t)|^2. \quad (30)$$

However, these relations have only been used to estimate the CKM matrix element, the form factors have been taken from theory. For example, lattice QCD has been used to estimate $\langle \pi^+ | \bar{u}_\alpha \gamma^\mu b_\alpha | \bar{B}^0 \rangle$ at large values of t where the pion is moving slowly. Various phenomenological forms, such as (23) which interpolate quite well through the calculated values, are then used to extrapolate to the small t region. Table 2 shows values for $F_\pi(0)$ and $F_K(0)$ from lattice QCD and other more phenomenological estimates. Other transition form factors are related by isospin symmetry, for example

$$\langle \pi^0 | \bar{u}_\alpha \gamma^\mu b_\alpha | B^- \rangle = \frac{1}{\sqrt{2}} \langle \pi^+ | \bar{u}_\alpha \gamma^\mu b_\alpha | \bar{B}^0 \rangle. \quad (31)$$

The transition matrix elements to vector mesons are usually expressed through four form factors:

$$\begin{aligned} \langle V | \bar{q}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha | B \rangle = & 2i\varepsilon_{\mu\nu\rho\sigma} \epsilon^\nu p_V^\rho p_B^\sigma \frac{V(t)}{m_B + m_V} \\ & - \epsilon^\mu (m_B + m_V) A_1(t) + (p_B + p_V)^\mu \epsilon \cdot q \frac{A_2(t)}{m_B + m_V} \\ & + q^\mu \epsilon \cdot q \frac{2m_V}{t} [A_3(t) - A_0(t)] \end{aligned} \quad (32)$$

where

$$A_3(t) = \left(\frac{m_B + m_V}{2m_V} \right) A_1(t) - \left(\frac{m_B - m_V}{2m_V} \right) A_2(t). \quad (33)$$

With an appropriate phase convention all the form factors can be taken to be real. Again the form factors are dimensionless analytic functions of t with the nearest singularity at t real and greater than m_B^2 . Also, the analytic structure demands that $A_3(0) = A_0(0)$. In the semileptonic decays the matrix elements for the vector mesons to have helicity $+1$, -1 or 0 , denoted by $H_+(t)$, $H_-(t)$ and $H_0(t)$ respectively, are given by [32]

$$\begin{aligned} H_\pm(t) &= (m_B + m_V) A_1(t) \mp \frac{2m_B |p_V|}{m_B + m_V} V(t), \\ H_0(t) &= \frac{1}{2m_V \sqrt{t}} \left[(m_B^2 - m_V^2 - t)(m_B + m_V) A_1(t) \frac{4m_B^2 |p_V|^2 A_2(t)}{m_B + m_V} \right] \end{aligned} \quad (34)$$

where the vector meson momentum $|p_V|$ in the B rest frame is

$$|p_V| = \left[\left(\frac{m_B^2 + m_V^2 - t}{2m_B} \right)^2 - m_V^2 \right]^{1/2}. \quad (35)$$

Note that $|p_V|$, $H_\pm(t)$ and $\sqrt{t}H_0(t)$ are analytic functions of t with singularities distant from $t = 0$. For small t it appears to be the case [32] that A_1 , A_2 and V are of similar magnitude. Hence, for $t \leq m_B^2$, and barring excessive cancellation, it can be anticipated that $\sqrt{t}H_0(t)/m_V$ will be larger than $H_\pm(t)$ by a factor of $(m_B/m_V)^2$.

The squares of some helicity matrix elements can in principle be measured from the semileptonic decays, for example, $\bar{B}^0 \rightarrow \rho^+ + l^- + \nu$. The decay rate for the lepton pair to have an invariant mass squared of t is

$$\frac{d\Gamma}{dt} = \frac{G_F^2}{256\pi^3} \frac{t |p_V| |V_{qb}|^2}{m_B^2} [H_+^2 (1 - \cos \theta)^2 + H_-^2 (1 + \cos \theta)^2 + 2H_0^2 \sin^2 \theta] \quad (36)$$

where θ is the angle between the charged lepton velocity and the recoil momentum of the vector boson $V = \rho^+$ in the lepton pair rest frame.

As with pseudoscalar transitions, these relations (36) have only been used to estimate the CKM matrix element $|V_{ub}|$, the form factors have been taken from theory such as lattice QCD. The CLEO collaboration [32] have made such an analysis with several theoretical models. All models but one show a substantial dominance of $H_0(t)$ at small t . Table 3 provides various theoretical estimates for the form factors and helicity matrix elements associated with the $\bar{B}^0 \rightarrow \rho^+$ and $B^- \rightarrow K^{*-}$ vector decays.

Contraction of the transition matrix element (32) with that for a pseudoscalar factorization partner gives

$$\langle P|J^\mu|0\rangle\langle V,\lambda=0|J_\mu|B\rangle = 2m_B|p_V|f_PA_0(m_P^2). \quad (37)$$

Here $\lambda = 0$ indicates that the vector meson must have zero helicity. For a vector factorization partner there are three possibilities. Since the B meson has no spin, both vector particles must have the same helicity so that

$$\langle V_2,\lambda|J^\mu|0\rangle\langle V_1,\lambda|J_\mu|B\rangle = f_{V_2}m_{V_2}H_\lambda(m_{V_2}^2) \quad (38)$$

where $\lambda = \pm 1, 0$. In the heavy meson limit both mesons should have zero helicity. Although some cancellation between form factors can be anticipated, Table 3 suggests that the helicity zero states will dominate the decay rates. Also

$$A_3(t) = \left(\frac{1}{m_B^2 - m_V^2 - t} \right) \left[\sqrt{t}H_0(t) - \frac{m_B^2 + 3m_V^2 - t}{2m_V(m_B + m_V)} A_2(t) \right] \quad (39)$$

so that, from (34), the constraint $A_3(0) = A_0(0)$ and the fact that the singularities are distant from the small t region, we can write, for example, in the heavy meson limit where terms in $(m_V/m_B)^2$ are neglected,

$$\langle \pi^- | (\bar{d}_\alpha u_\alpha)_{V-A} | 0 \rangle \langle V, 0 | (\bar{u}_\beta b_\beta)_{V-A} | B \rangle = f_\pi m_B^2 A_0(0) \quad (40)$$

and

$$\langle \rho^- | (\bar{d}_\alpha u_\alpha)_{V-A} | 0 \rangle \langle V, 0 | (\bar{u}_\beta b_\beta)_{V-A} | B \rangle = f_\rho m_B^2 A_0(0). \quad (41)$$

Thus the four soft QCD parameters characterizing decays into vector mesons reduce to just one in this limit.

Based upon the simplifications discussed above, we show in tables 4 to 7 our expressions for the factorization matrix elements

$$\tilde{Q}_i(h_1, h_2) \equiv m_B^{-2} [Q_i(h_1, h_2) + Q_i(h_2, h_1)] \quad (42)$$

for a wide range of B decays. In these tables we have used the notation $F_\pi = F_1^\pi(0)$, $A_\rho = A_0^\rho(0)$, etc. and include chiral enhancement factors $R_\chi^\pi = 2m_\pi^2/[m_b(m_u + m_d)]$ and $R_\chi^K = 2m_K^2/[m_b(m_u + m_s)]$ for the a_6 contributions.

4. Sign conventions for decay constants and form factors

It is clear in the literature (see, for example, references [9, 10, 32]) that different authors use different phase conventions for the particle states in defining the current matrix elements. Changes of convention should only multiply the combination $\sum_{i=1,\dots,6,9} p_i \tilde{Q}_i$ of the matrix elements \tilde{Q}_i occurring in (54) by a common phase factor, thus leaving the branching ratios unaltered. However, an inconsistent convention, such as defining f_π through $\langle \pi^+(p) | \bar{u}_\alpha \gamma_5 \gamma^\mu d_\alpha | 0 \rangle = f_\pi p_\pi^\mu$ but insisting that f_π is positive, will result in different branching ratios. If the experimental program, outlined in this paper, were to be carried through, the relative signs of these soft QCD parameters would not be determined and at this preliminary stage of B decay data analysis a theory must be

consulted to determine these signs. We outline here a simple quark model that illustrates this procedure.

Consider the current operator $j^\mu(0) = \bar{u}\gamma^\mu(1 - \gamma_5)d$ where the u and d quark fields are evaluated at $x^\mu = 0$. To construct the matrix elements $\langle\pi^+|j^\mu|0\rangle$ and $\langle\rho^+|j^\mu|0\rangle$ we take $|0\rangle$ to be the state with no quarks or antiquarks and $|\pi^+\rangle$ and $|\rho^+\rangle$ to be a $u\bar{d}$ pair at rest with a bound state S wave function $\phi(r)$ for their relative distance r . The π^+ and ρ^+ spin states are

$$|\pi^+\rangle_{\text{spin}} = \frac{1}{\sqrt{2}}[|u, \frac{1}{2}\rangle|\bar{d}, -\frac{1}{2}\rangle - |u, -\frac{1}{2}\rangle|\bar{d}, \frac{1}{2}\rangle] \quad (43)$$

and

$$|\rho^+\rangle_{\text{spin}} = \frac{1}{\sqrt{2}}[|u, \frac{1}{2}\rangle|\bar{d}, -\frac{1}{2}\rangle + |u, -\frac{1}{2}\rangle|\bar{d}, \frac{1}{2}\rangle]. \quad (44)$$

We then find, up to an overall positive dimensionless factor,

$$\langle\pi^+|j^\mu|0\rangle \propto \phi_\pi^*(0)[-1, 0, 0, 0] \quad (45)$$

and

$$\langle\rho^+|j^\mu|0\rangle \propto \phi_\rho^*(0)[0, 0, 0, 1]. \quad (46)$$

A comparison with (15), (18) and (27) then gives

$$f_\pi \propto \phi_\pi^*(0)/\sqrt{m_\pi} \quad (47)$$

and

$$f_\rho \propto \phi_\rho^*(0)/\sqrt{m_\rho}. \quad (48)$$

Similarly, we construct the matrix elements $\langle\pi^+|\bar{u}\gamma^\mu(1 - \gamma_5)b|\bar{B}^0\rangle$ and $\langle\rho^+|\bar{u}\gamma^\mu(1 - \gamma_5)b|\bar{B}^0\rangle$ by assuming that the \bar{B}^0 is at rest with a $b\bar{d}$ S wave function $\phi_B(r)$ and spin state

$$|\bar{B}^0\rangle_{\text{spin}} = \frac{1}{\sqrt{2}}[|b, \frac{1}{2}\rangle|\bar{d}, -\frac{1}{2}\rangle - |b, -\frac{1}{2}\rangle|\bar{d}, \frac{1}{2}\rangle]. \quad (49)$$

For the π^+ and ρ^+ again at rest we find, up to an overall positive factor,

$$\langle\pi^+|j^\mu|\bar{B}^0\rangle \propto \int \phi_\pi^*(r)\phi_B(r)r^2 dr [1, 0, 0, 0] \quad (50)$$

and

$$\langle\rho^+|j^\mu|\bar{B}^0\rangle \propto \int \phi_\rho^*(r)\phi_B(r)r^2 dr [0, 0, 0, -1] \quad (51)$$

so that, from (22), (32) and (27), we infer

$$F_0((m_B - m_\pi)^2) \propto \int \phi_\pi^*(r)\phi_B(r)r^2 dr \quad (52)$$

and

$$A_1((m_B - m_\rho)^2) \propto \int \phi_\rho^*(r)\phi_B(r)r^2 dr. \quad (53)$$

Noting that the form factors presented in the literature do not change sign on extrapolation to $t = 0$ then we observe that taking the wave functions to be all real and positive is in accord with our sign conventions for the four parameters f_π , f_ρ , F_0 and A_1 .

5. Confronting the model with data

The branching ratios for two-body B decays are given, in the heavy mass limit, by

$$\text{Br}(B \rightarrow h_1 h_2) = S \frac{G_F^2 m_B^3}{32\pi\Gamma_{\text{total}}} \left| \sum_{i=1,\dots,6,9} p_i \tilde{Q}_i(h_1, h_2) \right|^2 \quad (54)$$

where $S = 1$ unless the two bodies are identical, in which case the angular phase space is halved and $S = 1/2$. We have attempted to fit the theoretical expressions for branching ratios with available data. Measured branching ratios for nineteen channels are shown in table 8, along with references to where the data can be found. For many channels there are measurements presented by more than one of the three groups CLEO, BaBar and Belle. We have not attempted to combine the results, for each channel the table shows the measured branching ratio with the smallest quoted errors. We take the measured branching ratios to be the mean of the B and \bar{B} decays:

$$\text{Br}(\text{exp}) = \frac{1}{2} [\text{Br}(B \rightarrow h_1 h_2) + \text{Br}(\bar{B} \rightarrow \bar{h}_1 \bar{h}_2)]. \quad (55)$$

We ignore here the CP asymmetries

$$A_{\text{CP}} = \frac{\text{Br}(B \rightarrow h_1 h_2) - \text{Br}(\bar{B} \rightarrow \bar{h}_1 \bar{h}_2)}{\text{Br}(B \rightarrow h_1 h_2) + \text{Br}(\bar{B} \rightarrow \bar{h}_1 \bar{h}_2)} \quad (56)$$

which are, so far, all consistent with being zero.

For convenience we assign to each channel ($h_1 h_2$) a number α . The statistical and systematic errors have been combined into a single error σ_α . The systematic errors in particular can be expected to be correlated. Here we ignore all correlations and form a χ^2 function

$$\chi^2(P_i) = \sum_{\alpha} [|\text{Br}_\alpha(P_i) - \text{Br}_\alpha(\text{exp})|/\sigma_\alpha]^2 + \text{additional constraints}. \quad (57)$$

$\text{Br}_\alpha(P_i)$ are the theoretical branching ratios given by (54) in terms of nine parameters $P_i, i = 1, \dots, 9$ which we take to be the three Wolfenstein CKM parameters $\{A, \bar{\rho}, \bar{\eta}\}$ and the six soft QCD parameters $\{R_\chi^K, F_\pi, F_K, A_\rho, A_\omega, A_{K^*}\}$. The small contribution of the $b \rightarrow u$ penguin makes the branching ratios insensitive to R_χ^π , which we hold fixed at $R_\chi^\pi = 0.97$. The well known decay parameters $\{f_\pi, f_K, f_\rho, f_\omega, f_\phi, f_{K^*}\}$ are held at their mean values and the Wolfenstein CKM λ parameter is taken to be $\lambda = 0.2205$. Additional constraints were added to the χ^2 to take into account experimental and theoretical results lying outside the data on B decay branching ratios. For example, we took the Wolfenstein parameter A to be close to 0.802 as has been inferred from many weak decay measurements. We search for a minimum of χ^2 as a function of the P_i where the minimum is close to the expected values of the soft QCD parameters given in tables 2 and 3. We use the MINUIT [33] program to minimise the χ^2 .

The theoretical branching ratios and contributions of the individual channels to χ^2 based on these best fit values are given in table 8. The theoretical values shown use the a_i coefficients listed in table 1 as model 2. These are the process-independent generalized factorization a_i coefficients computed for the renormalization

scale $\mu = m_b/2$. Also in the fit we take the electroweak penguin contribution to be $a_9 = -0.0094 - 0.0002i$ [10]. Table 8 shows results with the systematic and statistical errors added in quadrature: $\sigma^2 = \sigma_{\text{stat}}^2 + \sigma_{\text{syst}}^2$. We have also done the computation using simple addition of errors $\sigma = \sigma_{\text{stat}} + \sigma_{\text{syst}}$. Both procedures are ad hoc, addition in quadrature reduces the influence of the systematic errors which are in general the smallest. The values of the best fit parameters $P_{2,i}$ are shown in table 9 together with our estimates of the two standard deviation errors. These errors are of course highly correlated. A plot of the error matrix ellipse for the Wolfenstein parameters $\bar{\rho}$ and $\bar{\eta}$ is shown in figure 1. The results in table 9 for the best fit values of the various form factors lie within the spread of theoretical estimates for these form factors (see tables 2 and 3).

6. Discussion and conclusions

We have investigated two-body charmless hadronic B decays within the so called QCD factorization model, making use of simplifications which arise from working in the heavy quark limit. This is particularly evident for the $B \rightarrow VV$ processes which we argue to be predominantly of zero helicity. Consequently a wide range of decays can be expressed in terms of a relatively small number of soft QCD parameters, thus providing a theoretical framework which should be adequate to confront data for some time to come.

Different factorization models merely modify the a_i coefficients which premultiply the various combinations of soft QCD parameters, thus allowing ready comparisons between these models.

If, in B decays to two vector mesons, there is a significant contribution from the $\lambda = \pm$ helicity states, it should be apparent in the Dalitz type plots for the final decay products. Table 3 suggests that the negative helicity state might be important for \bar{B}^0 and B^- decays, and the positive helicity state for B^0 and B^+ decays. Since each helicity contributes incoherently to the branching ratio, each helicity can be considered as a separate channel. The additional helicity channels can be included at the cost of extra soft QCD parameters. The only vector channels in table 8 are $B \rightarrow K^*\phi$ and in these channels there is some evidence [34] for contributions from non-zero helicities. It can be seen from tables 1, 6 and 7 that the zero helicity amplitudes (the only ones included) are proportional to A_{K^*} , a parameter which contributes significantly only to the $K^*\phi$ channels. Splitting the decay rates into the individual helicity channels will hardly effect the fit, it will only modify the estimate of A_{K^*} given in table 9 and introduce more soft QCD parameters for the other helicities.

To economize in the number of soft QCD parameters we have not included decay channels involving η and η' mesons. These amplitudes involve the mixing angle between the $(u\bar{u} + d\bar{d})$ and $s\bar{s}$ combinations. Also, in principle, there is mixing with $c\bar{c}$ which, though small, could make a significant contribution to decay modes through the enhanced quark decay modes $b \rightarrow cq\bar{c}$.

From table 8 it seems that of the nineteen channels included in the present analysis

only the PV channel $\pi^- K^{*0}$, and to a lesser extent $\pi^- K^{*+}$ and ωK^0 , give a large contribution to the overall χ^2 . The ωK^0 channel has its largest theoretical contribution from the $b \rightarrow s$ penguin and, in particular, from the a_4 and a_6 terms. The theoretical branching ratio is small because of the cancellation, evident in tables 1 and 6, between these terms. It is difficult for the theory to explain a branching ratio greater than 2×10^{-6} .

The $\pi^- K^{*0}$ channel is well measured, Belle gives a large branching ratio consistent with the BaBar result $(15.5 \pm 3.4 \pm 1.8) \times 10^{-6}$ shown in table 8, whereas the CLEO value is lower. It is interesting to compare this channel with $\pi^- K^0$, also well measured and with only a marginally larger branching ratio. A fit of the theoretical ratio

$$\frac{\text{Br}(B \rightarrow \pi^- K^0)}{\text{Br}(B \rightarrow \pi^- K^{*0})} = \left(\frac{f_K}{f_{K^*}} \right) \left(1 + R_\chi^K \frac{a_6}{a_4} \right)^2 \approx 0.547(1 + 1.3R_\chi^K)^2 \quad (58)$$

to the BaBar and Belle data implies $R_\chi^K \leq 0.5$. Our fit does agree well with the CLEO result.

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Tables and table captions

Table 1. Decay amplitude coefficients a_i in the matrix elements of the effective Hamiltonian (8) for several theoretical models. Model 1 is a simple tree plus QCD penguin as described in section 2, model 2 is generalized factorization with the effective Wilson coefficients evaluated at the renormalization scale $\mu = m_b/2$ and $q^2 = m_b^2/4$ [35], and model 3 is QCD factorization at $\mu = m_b/2$ evaluated using the expressions of Beneke *et al* [14, 23]. The imaginary part of each coefficient is in parentheses.

Model	a_1^u	a_2^u	$a_3^u = a_3^c$	a_4^u	a_4^c
1	1.0000	0.3333	0.0000	-0.0492 (-0.0234i)	-0.0524 (-0.0059i)
2	1.0523	0.0333	0.0062	-0.0397 (-0.0148i)	-0.0496 (-0.0094i)
3	1.0510 (0.0335i)	0.0499 (-0.1063i)	0.0050 (0.0033i)	-0.030 (-0.019i)	-0.038 (-0.009i)
	$a_5^u = a_5^c$	a_6^u	a_6^c		
1	0.0000	-0.0492 (-0.0234i)	-0.0524 (-0.0059i)		
2	-0.0064	-0.0397 (-0.0148i)	-0.0496 (-0.0094i)		
3	0.0050 (0.0033i)	-0.050 (-0.017i)	-0.055 (-0.005i)		

Table 2. Theoretical form factors for $B \rightarrow \pi$ and $B \rightarrow K$ transitions.

Model	$F_\pi(0)$	$F_K(0)$
Lattice QCD ^a	0.27	
Quark model wave functions ^b	0.33	
Quark model wave functions ^c		0.38
Light cone sum rule ^d	0.305	0.341

^a Ref. [36]

^b Ref. [37]

^c Ref. [38]

^d Ref. [39]

Table 3. Theoretical form factors and helicity amplitudes for $\bar{B}^0 \rightarrow \rho^+$ and $B^- \rightarrow K^{*-}$ transitions. It can be seen that, although the helicity zero channel for $B \rightarrow VV$ decays can be expected to dominate, this table shows that much cancellation is anticipated in (34). The extent to which the helicity zero states dominate should be apparent in Dalitz type plots for the final decay products.

Reference	$A_1(0)$	$A_2(0)$	$V(0)$	$A_0(0)$	$ H_+(m_V^2) ^2$	$ H_-(m_V^2) ^2$	$ H_0(m_V^2) ^2$
$\bar{B}^0 \rightarrow \rho^+$							
1 ^a	0.27	0.26	0.35	0.30	0.008	10.1	121
2 ^b	0.26	0.22	0.34	0.38	0.006	9.4	185
3 ^c	0.27	0.28	0.35	0.24	0.008	10.1	81
4 ^d	0.30	0.33	0.37	0.21	0.034	11.9	66
5 ^e	0.28	0.28	0.33	0.28	0.057	9.9	107
$B^- \rightarrow K^{*-}$							
2 ^b	0.34	0.28	0.46	0.49	0.02	16.4	224
4 ^d	0.36	0.40	0.45	0.26	0.10	17.1	75
6 ^f	0.37	0.40	0.47	0.30	0.30	18.3	93
7 ^g	0.33	0.33	0.37	0.32	0.22	13.0	110

^a Ref. [36] ^b Ref. [41] ^c Ref. [42] ^d Ref. [43] ^e Ref. [37] ^f Ref. [40] ^g Ref. [38]

Table 4. Factorization matrix elements $\tilde{Q}_i(h_1, h_2)$ for $\bar{B}^0 \rightarrow h_1 h_2$ decays arising from $b \rightarrow dq\bar{q}$.

Decay ^a	\tilde{Q}_1	\tilde{Q}_2	\tilde{Q}_3	\tilde{Q}_4	\tilde{Q}_5	\tilde{Q}_6	\tilde{Q}_9
$\pi^+\pi^-$	$-F_\pi f_\pi$	0	0	$-F_\pi f_\pi$	0	$-R_\chi^\pi F_\pi f_\pi$	0
$\pi^0\pi^0$	0	$F_\pi f_\pi$	0	$-F_\pi f_\pi$	0	$-R_\chi^\pi F_\pi f_\pi$	$\frac{3F_\pi f_\pi}{2}$
$\rho^+\pi^-$	$A_\rho f_\pi$	0	0	$A_\rho f_\pi$	0	$-R_\chi^\pi A_\rho f_\pi$	0
$\rho^0\pi^0$	0	$-\frac{A_\rho f_\pi + F_\pi f_\rho}{2}$	0	$\frac{A_\rho f_\pi + F_\pi f_\rho}{2}$	0	$-R_\chi^\pi \frac{A_\rho f_\pi}{2}$	$-\frac{3(A_\rho f_\pi + F_\pi f_\rho)}{4}$
$\pi^+\rho^-$	$F_\pi f_\rho$	0	0	$F_\pi f_\rho$	0	0	0
$\omega\pi^0$	0	$\frac{A_\omega f_\pi - F_\pi f_\omega}{2}$	$-F_\pi f_\omega$	$-\frac{A_\omega f_\pi + F_\pi f_\omega}{2}$	$-F_\pi f_\omega$	$R_\chi^\pi \frac{A_\omega f_\pi}{2}$	$\frac{3A_\omega f_\pi - F_\pi f_\omega}{4}$
$\rho^+\rho^-$	$A_\rho f_\rho$	0	0	$A_\rho f_\rho$	0	0	0
$\rho^0\rho^0$	0	$-A_\rho f_\rho$	0	$A_\rho f_\rho$	0	0	$-\frac{3A_\rho f_\rho}{2}$
$\omega\rho^0$	0	$\frac{A_\omega f_\rho - A_\rho f_\omega}{2}$	$-A_\rho f_\omega$	$-\frac{A_\omega f_\rho + A_\rho f_\omega}{2}$	$-A_\rho f_\omega$	0	$\frac{3A_\omega f_\rho - A_\rho f_\omega}{4}$
$\omega\omega$	0	$A_\omega f_\omega$	$2A_\omega f_\omega$	$A_\omega f_\omega$	$2A_\omega f_\omega$	0	$\frac{A_\omega f_\omega}{2}$
$\bar{K}^0 K^0$	0	0	0	$-F_K f_K$	0	$-R_\chi^K F_K f_K$	0
$\bar{K}^{*0} K^0$	0	0	0	$A_{K^*} f_K$	0	$-R_\chi^K A_{K^*} f_K$	0
$\bar{K}^0 K^{*0}$	0	0	0	$F_K f_{K^*}$	0	0	0
$\bar{K}^{*0} K^{*0}$	0	0	0	$A_{K^*} f_{K^*}$	0	0	0
$\phi\pi^0$	0	0	$\frac{F_\pi f_\phi}{\sqrt{2}}$	0	$\frac{F_\pi f_\phi}{\sqrt{2}}$	0	$-\frac{F_\pi f_\phi}{3\sqrt{2}}$

^a The decays to $\phi\rho^0$ and $\phi\omega$ are obtained from $\phi\pi^0$ by the substitutions $F_\pi \rightarrow A_\rho$ and $F_\pi \rightarrow -A_\omega$ respectively. The decays to K^+K^- , K^+K^{*-} , $K^{*+}K^-$ and $K^{*+}K^{*-}$ receive no contribution from $\tilde{Q}_{1,\dots,6}$

Table 5. Factorization matrix elements $\tilde{Q}_i(h_1, h_2)$ for $B^- \rightarrow h_1 h_2$ decays arising from $b \rightarrow dq\bar{q}$.

Decay ^a	\tilde{Q}_1	\tilde{Q}_2	\tilde{Q}_3	\tilde{Q}_4	\tilde{Q}_5	\tilde{Q}_6	\tilde{Q}_9
$\pi^0 \pi^-$	$-\frac{F_\pi f_\pi}{\sqrt{2}}$	$-\frac{F_\pi f_\pi}{\sqrt{2}}$	0	0	0	0	$-\frac{3F_\pi f_\pi}{2\sqrt{2}}$
$\rho^0 \pi^-$	$\frac{A_\rho f_\pi}{\sqrt{2}}$	$\frac{F_\pi f_\rho}{\sqrt{2}}$	0	$\frac{A_\rho f_\pi - F_\pi f_\rho}{\sqrt{2}}$	0	$-R_\chi^\pi \frac{A_\rho f_\pi}{\sqrt{2}}$	$\frac{3F_\pi f_\rho}{2\sqrt{2}}$
$\omega \pi^-$	$\frac{A_\omega f_\pi}{\sqrt{2}}$	$\frac{F_\pi f_\omega}{\sqrt{2}}$	$\sqrt{2} F_\pi f_\omega$	$\frac{A_\omega f_\pi + F_\pi f_\omega}{\sqrt{2}}$	$\sqrt{2} F_\pi f_\omega$	$-R_\chi^\pi \frac{A_\omega f_\pi}{\sqrt{2}}$	$\frac{F_\pi f_\omega}{2\sqrt{2}}$
$\pi^0 \rho^-$	$\frac{F_\pi f_\rho}{\sqrt{2}}$	$\frac{A_\rho f_\pi}{\sqrt{2}}$	0	$\frac{F_\pi f_\rho - A_\rho f_\pi}{\sqrt{2}}$	0	$R_\chi^\pi \frac{A_\rho f_\pi}{\sqrt{2}}$	$\frac{3A_\rho f_\pi}{2\sqrt{2}}$
$\rho^0 \rho^-$	$\frac{A_\rho f_\rho}{\sqrt{2}}$	$\frac{A_\rho f_\rho}{\sqrt{2}}$	0	0	0	0	$\frac{3A_\rho f_\rho}{2\sqrt{2}}$
$\omega \rho^-$	$\frac{A_\omega f_\rho}{\sqrt{2}}$	$\frac{A_\rho f_\omega}{\sqrt{2}}$	$\sqrt{2} A_\rho f_\omega$	$\frac{A_\omega f_\rho + A_\rho f_\omega}{\sqrt{2}}$	$\sqrt{2} A_\rho f_\omega$	0	$\frac{A_\rho f_\omega}{2\sqrt{2}}$
$K^- K^0$	0	0	0	$-F_K f_K$	0	$-R_\chi^K F_K f_K$	0
$K^{*-} K^0$	0	0	0	$A_{K^*} f_K$	0	$-R_\chi^K A_{K^*} f_K$	0
$K^- K^{*0}$	0	0	0	$F_K f_{K^*}$	0	0	0
$K^{*-} K^{*0}$	0	0	0	$A_{K^*} f_{K^*}$	0	0	0
$\phi \pi^-$	0	0	$F_\pi f_\phi$	0	$F_\pi f_\phi$	0	$-\frac{F_\pi f_\phi}{2}$

^a The decay to $\phi \rho^-$ is obtained from that to $\phi \pi^-$ by the substitution $F_\pi \rightarrow A_\rho$.

Table 6. Factorization matrix elements $\tilde{Q}_i(h_1, h_2)$ for $\bar{B}^0 \rightarrow h_1 h_2$ decays arising from $b \rightarrow sq\bar{q}$.

Decay	\tilde{Q}_1	\tilde{Q}_2	\tilde{Q}_3	\tilde{Q}_4	\tilde{Q}_5	\tilde{Q}_6	\tilde{Q}_9
$K^- \pi^+$	$-F_\pi f_K$	0	0	$-F_\pi f_K$	0	$-R_\chi^K F_\pi f_K$	0
$K^{*-} \pi^+$	$F_\pi f_{K^*}$	0	0	$F_\pi f_{K^*}$	0	0	0
$K^- \rho^+$	$A_\rho f_K$	0	0	$A_\rho f_K$	0	$-R_\chi^K A_\rho f_K$	0
$K^{*-} \rho^+$	$A_\rho f_{K^*}$	0	0	$A_\rho f_{K^*}$	0	0	0
$\bar{K}^0 \pi^0$	0	$-\frac{F_K f_\pi}{\sqrt{2}}$	0	$\frac{F_\pi f_K}{\sqrt{2}}$	0	$R_\chi^K \frac{F_\pi f_K}{\sqrt{2}}$	$-\frac{3F_K f_\pi}{2\sqrt{2}}$
$\bar{K}^{*0} \pi^0$	0	$\frac{A_{K^*} f_\pi}{\sqrt{2}}$	0	$-\frac{F_\pi f_{K^*}}{\sqrt{2}}$	0	0	$\frac{3A_{K^*} f_\pi}{2\sqrt{2}}$
$\bar{K}^0 \rho^0$	0	$\frac{F_K f_\rho}{\sqrt{2}}$	0	$-\frac{A_\rho f_K}{\sqrt{2}}$	0	$R_\chi^K \frac{A_\rho f_K}{\sqrt{2}}$	$\frac{3F_K f_\rho}{2\sqrt{2}}$
$\bar{K}^{*0} \rho^0$	0	$\frac{A_{K^*} f_\rho}{\sqrt{2}}$	0	$-\frac{A_\rho f_{K^*}}{\sqrt{2}}$	0	0	$\frac{3A_{K^*} f_\rho}{2\sqrt{2}}$
$\bar{K}^0 \omega$	0	$\frac{F_K f_\omega}{\sqrt{2}}$	$\sqrt{2} F_K f_\omega$	$\frac{A_\omega f_K}{\sqrt{2}}$	$\sqrt{2} F_K f_\omega$	$-R_\chi^K \frac{A_\omega f_K}{\sqrt{2}}$	$\frac{F_K f_\omega}{2\sqrt{2}}$
$\bar{K}^{*0} \omega$	0	$\frac{A_{K^*} f_\omega}{\sqrt{2}}$	$\sqrt{2} A_{K^*} f_\omega$	$\frac{A_\omega f_{K^*}}{\sqrt{2}}$	$\sqrt{2} A_{K^*} f_\omega$	0	$\frac{A_{K^*} f_\omega}{2\sqrt{2}}$
$\bar{K}^0 \phi$	0	0	$F_K f_\phi$	$F_K f_\phi$	$F_K f_\phi$	0	$-\frac{F_K f_\phi}{2}$
$\bar{K}^{*0} \phi$	0	0	$A_{K^*} f_\phi$	$A_{K^*} f_\phi$	$A_{K^*} f_\phi$	0	$-\frac{A_{K^*} f_\phi}{2}$

Table 7. Factorization matrix elements $\tilde{Q}_i(h_1, h_2)$ for $B^- \rightarrow h_1 h_2$ decays arising from $b \rightarrow sq\bar{q}$.

Decay	\tilde{Q}_1	\tilde{Q}_2	\tilde{Q}_3	\tilde{Q}_4	\tilde{Q}_5	\tilde{Q}_6	\tilde{Q}_9
$\pi^0 K^-$	$-\frac{F_\pi f_K}{\sqrt{2}}$	$-\frac{F_K f_\pi}{\sqrt{2}}$	0	$-\frac{F_\pi f_K}{\sqrt{2}}$	0	$-R_\chi^K \frac{F_\pi f_K}{\sqrt{2}}$	$-\frac{3F_K f_\pi}{2\sqrt{2}}$
$\pi^0 K^{*-}$	$\frac{F_\pi f_{K^*}}{\sqrt{2}}$	$\frac{A_{K^*} f_\pi}{\sqrt{2}}$	0	$\frac{F_\pi f_{K^*}}{\sqrt{2}}$	0	0	$\frac{3A_{K^*} f_\pi}{2\sqrt{2}}$
$\pi^- \bar{K}^0$	0	0	0	$-F_\pi f_K$	0	$-R_\chi^K F_\pi f_K$	0
$\pi^- \bar{K}^{*0}$	0	0	0	$F_\pi f_{K^*}$	0	0	0
$\rho^- \bar{K}^0$	0	0	0	$A_\rho f_K$	0	$-R_\chi^K A_\rho f_K$	0
$\rho^- \bar{K}^{*0}$	0	0	0	$A_\rho f_{K^*}$	0	0	0
$\rho^0 K^-$	$\frac{A_\rho f_K}{\sqrt{2}}$	$\frac{F_K f_\rho}{\sqrt{2}}$	0	$\frac{A_\rho f_K}{\sqrt{2}}$	0	$-R_\chi^K \frac{A_\rho f_K}{\sqrt{2}}$	$\frac{3F_K f_\rho}{2\sqrt{2}}$
$\rho^0 K^{*-}$	$\frac{A_\rho f_{K^*}}{\sqrt{2}}$	$\frac{A_{K^*} f_\rho}{\sqrt{2}}$	0	$\frac{A_\rho f_{K^*}}{\sqrt{2}}$	0	0	$\frac{3A_{K^*} f_\rho}{2\sqrt{2}}$
ωK^-	$\frac{A_\omega f_K}{\sqrt{2}}$	$\frac{F_K f_\omega}{\sqrt{2}}$	$\sqrt{2} F_K f_\omega$	$\frac{A_\omega f_K}{\sqrt{2}}$	$\sqrt{2} F_K f_\omega$	$-R_\chi^K \frac{A_\omega f_K}{\sqrt{2}}$	$\frac{F_K f_\omega}{2\sqrt{2}}$
ωK^{*-}	$\frac{A_\omega f_{K^*}}{\sqrt{2}}$	$\frac{A_{K^*} f_\omega}{\sqrt{2}}$	$\sqrt{2} A_{K^*} f_\omega$	$\frac{A_\omega f_{K^*}}{\sqrt{2}}$	$\sqrt{2} A_{K^*} f_\omega$	0	$\frac{A_{K^*} f_\omega}{2\sqrt{2}}$
ϕK^-	0	0	$F_K f_\phi$	$F_K f_\phi$	$F_K f_\phi$	0	$-\frac{F_K f_\phi}{2}$
ϕK^{*-}	0	0	$A_{K^*} f_\phi$	$A_{K^*} f_\phi$	$A_{K^*} f_\phi$	0	$-\frac{A_{K^*} f_\phi}{2}$

Table 8. Measured branching ratio $\text{Br}(\text{exp})$, experimental error σ (errors added in quadrature), theoretical branching ratio for best fit parameters $\text{Br}(\text{fit})$ and contribution to χ^2 for various B decay channels.

Decay	$\text{Br}(\text{exp})^a$	σ^a	Reference ^b	$\text{Br}(\text{fit})^a$	χ^2
$\pi^+\pi^-$	4.1	1.2	Ba1,Be1,Cl1	5.4	1.07
$\rho^\pm\pi^\mp$	28.9	6.9	Ba2,Be2,Cl1	29.6	0.01
$\rho^0\pi^0$	3.6	3.9	Ba2	0.1	0.15
$\omega\pi^0$	0.8	2.0	Ba3,Cl2	0.1	0.15
$\rho^0\pi^-$	10.4	3.9	Be2,Cl1	9.0	0.13
$\omega\pi^-$	6.6	2.2	Ba3,Cl2	7.15	0.06
π^0K^-	10.8	2.4	Ba1,Be1,Cl1	11.1	0.02
π^-K^0	18.2	3.9	Ba1,Be1, Cl1	18.1	0.00
π^-K^{*0}	15.9	3.7	Ba4,Be3,Cl2	5.0	8.75
ωK^-	3.2	2.2	Cl2	1.4	0.68
ϕK^-	7.7	1.8	Ba5,Be2,Cl1	7.8	0.00
ϕK^{*-}	9.6	4.4	Ba5, Cl1	9.6	0.00
π^-K^+	16.7	2.2	Ba1, Be1,Cl1	16.9	0.01
π^-K^{*+}	22	11	Cl2	5.0	2.39
π^0K^0	8.2	3.3	Ba1,Be1,Cl1	6.7	0.20
π^0K^{*0}	2.1	2.1	Cl2	1.4	0.12
ωK^0	10	6	Cl2	0.7	2.42
ϕK^0	8.1	3.2	Ba5	7.4	0.05
ϕK^{*0}	8.6	3.0	Ba5, Be2, Cl1	9.0	0.02

^a In units of 10^{-6} .^b References to experimental groups are: BaBar Ba1 Ref. [44], Ba2 Ref. [45], Ba3 Ref. [46], Ba4 Ref. [47], Ba5 Ref. [4]; Belle Be1 Ref. [48], Be2 Ref. [49], Be3 Ref. [50] and CLEO Cl1 Ref. [51], Cl2 Ref. [52].**Table 9.** Best fit values and one-standard deviation errors of fitting parameters.

F_π	F_K	A_ρ	A_ω	A_{K^*}
0.243 ± 0.031	0.312 ± 0.040	0.404 ± 0.087	0.377 ± 0.067	0.349 ± 0.052
R_χ^π	R_χ^K	A	$\bar{\eta}$	$\bar{\rho}$
0.970 ± 0.000	1.200 ± 0.186	0.803 ± 0.063	0.375 ± 0.053	0.038 ± 0.104

Figure captions

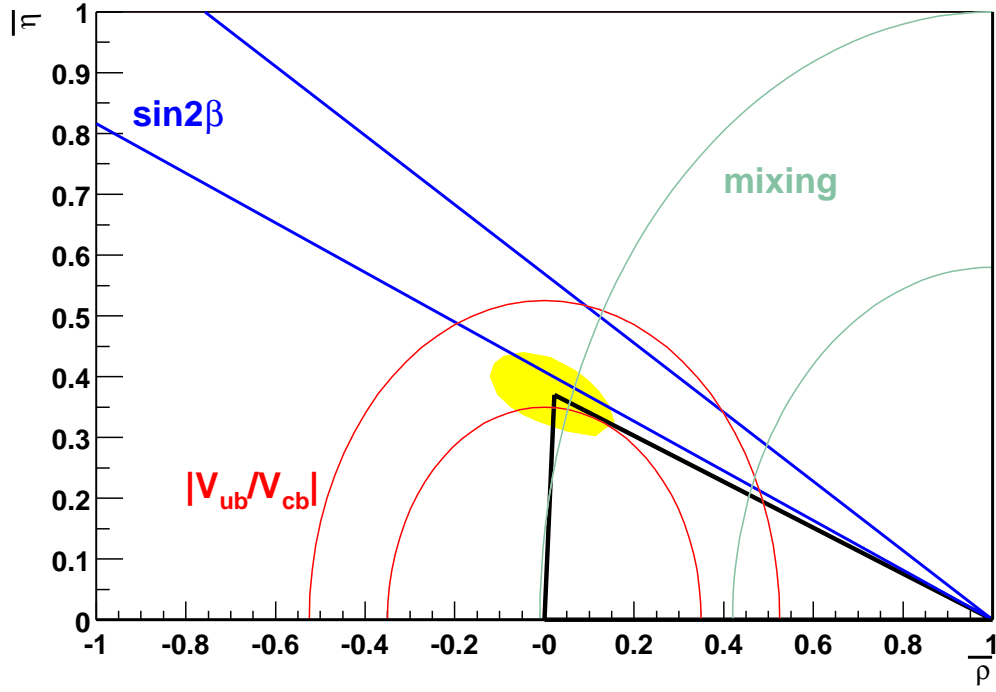


Figure 1. The 95% CL error ellipse for CKM parameters $\bar{\rho}$ and $\bar{\eta}$ together with the constraints on the Unitarity Triangle from mixing, $|V_{ub}/V_{cb}|$ and $\sin 2\beta$.